

$$[2] f(t) = \begin{cases} 2t, & 0 < t < 1 \\ 3-t, & t > 1 \end{cases} \quad (5) = 2t + u(t-1)(3-t-2t) \quad (5)$$

$$\mathcal{L}\{f(t)\} = \frac{2}{s^2} + e^{-s} \mathcal{L}\{3-3(t+1)\}$$

$$= \frac{2}{s^2} + e^{-s} \mathcal{L}\{-3t\} \quad (5)$$

$$= \left[\frac{2}{s^2} - e^{-s} \frac{3}{s^2} \right] \quad (5)$$

$$(5) \quad s^2 Y - s y(0) - y'(0) + 4Y = \frac{2}{s^2} - e^{-s} \frac{3}{s^2}$$

$$(5) \quad (s^2 + 4)Y = 2s - 1 + \frac{2}{s^2} - e^{-s} \frac{3}{s^2}$$

$$Y = \left[\frac{2s-1}{s^2+4} \right] + \left[2 \cdot \frac{1}{s^2(s^2+4)} - 3e^{-s} \frac{1}{s^2(s^2+4)} \right] \quad (2\frac{1}{2})$$

$$= 2 \cdot \frac{s}{s^2+4} - \frac{1}{2} \cdot \frac{2}{s^2+4} + 2 \left(\frac{1}{4} \cdot \frac{1}{s^2} - \frac{1}{8} \cdot \frac{2}{s^2+4} \right) \quad (10)$$

$$- 3e^{-s} \left(\frac{1}{4} \cdot \frac{1}{s^2} - \frac{1}{8} \cdot \frac{2}{s^2+4} \right)$$

$$Y = \left[2\cos 2t - \frac{1}{2}\sin 2t \right] + \left[2\left(\frac{1}{4}t - \frac{1}{8}\sin 2t\right) \right] \quad (5)$$

$$(7\frac{1}{2}) \quad - 3u(t-1)\left(\frac{1}{4}(t-1) - \frac{1}{8}\sin 2(t-1)\right)$$

$$= \left[2\cos 2t - \frac{3}{4}\sin 2t + \frac{1}{2}t \right. \\ \left. + u(t-1)\left(\frac{3}{4} - \frac{3}{4}t + \frac{3}{8}\sin 2(t-1)\right) \right] \quad (2\frac{1}{2})$$

$$(5) \quad \frac{1}{s^2(s^2+4)} = \frac{0}{s} + \frac{\frac{1}{4}}{s^2} + \frac{0}{s^2+4} + \frac{-\frac{1}{8}}{s^2+4} \quad (5) \rightarrow 1 = As(s^2+4) + B(s^2+4) + Cs^3 + D(2s^2)$$

$$s=0: 1 = 4B \rightarrow B = \frac{1}{4}$$

$$s^2: 0 = B + 2D \rightarrow D = -\frac{1}{2}B = -\frac{1}{8}$$

$$s: 0 = 4A \rightarrow A = 0$$

$$s^3: 0 = A + C \rightarrow C = -A = 0$$

SANITY CHECK $s=2$

$$\frac{1}{4(8)} = \frac{1}{32} \quad (5)$$

$$\frac{\frac{1}{4}}{4} - \frac{\frac{1}{4}}{8} = \frac{1}{16} - \frac{1}{32} = \frac{1}{32} \quad \checkmark$$

$$[3] f(t) = \begin{cases} t, & 0 < t < 2 \\ 6-2t, & 2 < t < 3 \\ 0, & t > 3 \end{cases} = t + u(t-2)(6-2t-t) + u(t-3)(0-(6-2t))$$

$$\textcircled{4} = t + u(t-2)(6-3t) + u(t-3)(2t-6)$$

$$\mathcal{L}\{f_T\} = \frac{1}{s^2} + e^{-2s} \mathcal{L}\{6-3(t+2)\} + e^{-3s} \mathcal{L}\{2(t+3)-6\} \quad \textcircled{4}$$

$$= \frac{1}{s^2} + e^{-2s} \mathcal{L}\{-3t\} + e^{-3s} \mathcal{L}\{2t\} \quad \textcircled{4}$$

$$= \frac{1}{s^2} - 3e^{-2s} \frac{1}{s^2} + 2e^{-3s} \frac{1}{s^2} \quad \textcircled{4}$$

$$\mathcal{L}\{f\} = \frac{\frac{1}{s^2} - 3e^{-2s} \frac{1}{s^2} + 2e^{-3s} \frac{1}{s^2}}{1 - e^{-3s}} = F(s) \quad \textcircled{4}$$

$$\textcircled{4} s^2 Y - s y(0) - y'(0) + 3(sY - y(0)) + 2Y = F(s)$$

$$\textcircled{4} (s^2 + 3s + 2)Y = -s - 1 + F(s)$$

$$Y = \frac{-s-1}{(s+1)(s+2)} + \frac{1}{s^2(s+1)(s+2)} - 3e^{-2s} \frac{1}{s^2(s+1)(s+2)} + 2e^{-3s} \frac{1}{s^2(s+1)(s+2)} \quad \textcircled{4}$$

$$\frac{1}{s^2(s+1)(s+2)}$$

$$\textcircled{4} = -\frac{1}{s+2} + \frac{1}{1-e^{-3s}} \left(\frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{1}{s+1} + \frac{-\frac{1}{4}}{s+2} \right) \quad \textcircled{16}$$

$$- \frac{3e^{-2s}}{1-e^{-3s}} \left(\frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{1}{s+1} + \frac{-\frac{1}{4}}{s+2} \right)$$

$$+ \frac{2e^{-3s}}{1-e^{-3s}} \left(\frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{1}{s+1} + \frac{-\frac{1}{4}}{s+2} \right)$$

$$= -\frac{1}{s+2} + (1 + e^{-3s} + e^{-6s} + \dots) \left(\frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{1}{s+1} + \frac{-\frac{1}{4}}{s+2} \right)$$

$$\textcircled{4} - 3(e^{-2s} + e^{-5s} + e^{-8s} + \dots) \left(\frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{1}{s+1} + \frac{-\frac{1}{4}}{s+2} \right)$$

$$\textcircled{4} + 2(e^{-3s} + e^{-6s} + e^{-9s} + \dots) \left(\frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{1}{s+1} + \frac{-\frac{1}{4}}{s+2} \right)$$

$$= -\frac{\frac{5}{4}}{s+2} + \frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{1}{s+1}$$

$$\textcircled{4} + 3(e^{-2s} + e^{-6s} + e^{-9s} + \dots) \left(\frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{1}{s+1} + \frac{-\frac{1}{4}}{s+2} \right)$$

$$- 3(e^{-2s} + e^{-5s} + e^{-8s} + \dots) \left(\frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{1}{s+1} + \frac{-\frac{1}{4}}{s+2} \right)$$

$$\boxed{\frac{1}{s^2(s+1)(s+2)} = \frac{A^{-\frac{3}{4}}}{s} + \frac{B^{\frac{1}{2}}}{s^2} + \frac{C^1}{s+1} + \frac{D^{-\frac{1}{4}}}{s+2}} \quad (4)$$

$$\boxed{1 = As(s+1)(s+2) + B(s+1)(s+2) + Cs^2(s+2) + Ds^2(s+1)}$$

$$s=0: 1 = 2B \rightarrow B = \frac{1}{2} \quad (4)$$

$$s=-1: 1 = C$$

$$s=-2: 1 = -4D \rightarrow D = -\frac{1}{4}$$

$$s^3: 0 = A + C + D \rightarrow A = -C - D = -\frac{3}{4}$$

SANITY CHECK $s=2$

$$\boxed{\begin{aligned} \frac{1}{4(3)(4)} &= \frac{1}{48} \\ -\frac{\frac{3}{4}}{2} + \frac{\frac{1}{2}}{4} + \frac{1}{3} - \frac{\frac{1}{4}}{4} &= -\frac{3}{8} + \frac{1}{8} + \frac{1}{3} - \frac{1}{16} \\ &= \frac{-18+6+16-3}{48} = \frac{1}{48} \checkmark \end{aligned}} \quad (4)$$

$$\mathcal{L}^{-1}\left\{\frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{1}{s+1} + \frac{-\frac{1}{4}}{s+2}\right\} = \boxed{-\frac{3}{4} + \frac{1}{2}t + e^{-t} - \frac{1}{4}e^{-2t}} \quad (4)$$

$$\mathcal{L}^{-1}\left\{e^{-ns}\left(\frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{1}{s+1} + \frac{-\frac{1}{4}}{s+2}\right)\right\}$$

$$= u(t-n)\left(-\frac{3}{4} + \frac{1}{2}(t-n) + e^{-(t-n)} - \frac{1}{4}e^{-2(t-n)}\right)$$

$$y = \boxed{-\frac{5}{4}e^{-2t} - \frac{3}{4} + \frac{1}{2}t + e^{-t}} \quad (4)$$

$$(4) + \sum_{n=1}^{\infty} u(t-3n)\left(-\frac{9}{4} + \frac{3}{2}(t-3n) + 3e^{-(t-3n)} - \frac{3}{4}e^{-2(t-3n)}\right)$$

$$(4) + \sum_{n=1}^{\infty} u(t-(3n-1))\left(\frac{9}{4} - \frac{3}{2}(t-(3n-1)) - 3e^{-(t-(3n-1))} + \frac{3}{4}e^{-2(t-(3n-1))}\right)$$